^{7}Li Abundances in Halo Stars:
Testing Stellar Evolution Models
and the Primordial ^{7}Li Abundance

Brian Chaboyer[1,2] and P. Demarque[1,3]

[1]Department of Astronomy, and Center for Solar and Space Research, Yale University, Box 208101, New Haven, CT 06520-8101

[2]present address: CITA, 60 St. George St., U. of Toronto, Toronto, Ontario, Canada M5S 1A7 Electronic Mail – I:chaboyer@cita.utoronto.ca

[3]Center for Theoretical Physics, Yale University
Electronic Mail –I:demarque@astro.yale.edu

ABSTRACT

A large number of stellar evolution models with [Fe/H] $= -2.3$ and -3.3 have been calculated in order to determine the primordial ^{7}Li abundance and to test current stellar evolution models by a comparison to the extensive database of accurate Li abundances in extremely metal poor halo stars observed by Thorburn (1994). Standard models with grey atmospheres do a very good job of fitting the observed Li abundances in stars hotter than ~ 5600 K. They predict a primordial ^{7}Li abundance of $\log N(\mathrm{Li}) = 2.24 \pm 0.03$. Models which include microscopic diffusion predict a downward curvature in the ^{7}Li destruction isochrones at hot temperatures which is not present in the observations. Thus, the observations clearly rule out models which include uninhibited microscopic diffusion of ^{7}Li from the surface of the star. Rotational mixing inhibits the microscopic diffusion and the [Fe/H] $= -2.28$ stellar models which include both diffusion and rotational mixing provide an excellent match to the mean trend in T_{eff} which is present in the observations. Both the plateau stars and the heavily depleted cool stars are well fit by these models. The rotational mixing leads to considerable ^{7}Li depletion in these models and the primordial ^{7}Li abundance inferred from these models is $\log N(\mathrm{Li}) = 3.08 \pm 0.1$. However, the [Fe/H] $= -3.28$ isochrones reveal problems with the combined models. These isochrones predict a trend of decreasing $\log N(\mathrm{Li})$ with increasing T_{eff} which is not present in the observations. Possible causes for this discrepancy are discussed.

Subject headings: cosmology: early universe – nucleosynthesis – stars: interiors – stars: abundances

1. Introduction

Standard big bang nucleosynthesis (BBN) has been remarkably successfully in predicting the abundance of the various light elements (^{1}H, ^{2}H, ^{3}He, ^{4}He, ^{7}Li). In standard BBN, consistency with the inferred primordial abundance of ^{2}H, ^{3}He and ^{4}He requires the primordial ^{7}Li abundance in the range $1.9 < \log N(\text{Li}) < 2.3$ (Krauss & Romanelli 1990; Walker *et al.* 1991; Smith Kawano & Malaney 1993). In recent years, various alternatives to the standard big bang have been proposed (cf. the review by Malaney & Mathews 1993). Perhaps the most important of these are the inhomogeneous theories, which assume that density inhomogeneities exist prior to nucleosynthesis (Witten 1984). In general, inhomogeneous BBN models produce far more ^{7}Li than standard BBN. Thus, the primordial ^{7}Li abundance provides an important test of BBN. Observations of Li[4] in old, metal poor hot stars ([Fe/H] $\lesssim -1.5$, $T_{\text{eff}} > 5590$ K) in our Galaxy have found a nearly uniform abundance of $\log N(\text{Li}) \simeq 2.2$ observed in hot stars (Spite & Spite 1982, 1986; Rebolo, Molaro & Beckman 1988; Hobbs & Thorburn 1991). This has been viewed as confirmation of standard BBN.

Unfortunately, this neat match between observation and theory has been brought into question by observations of extremely metal-poor stars ([Fe/H] < -1.9). Over 70 such stars have been observed (Thorburn 1994) and 3 of them show a significant ^{7}Li depletion. Thus, some mechanism is destroying ^{7}Li in at least some metal-poor stars. This may be taken as a sign that a few stars undergo an very efficient ^{7}Li depletion process, such as a stellar merger, which does not affect the large majority of plateau stars (Hobbs, Welty, & Thorburn 1991; Spite *et al.* 1993). However, there is convincing evidence from studies of young cluster stars that ^{7}Li depletion occurs on the main sequence in metal rich stars ([Fe/H] > -0.1) (Thorburn *et al.* 1993; Soderblom *et al.* 1993; Chaboyer, Demarque & Pinsonneault 1994a; 1994b, hereafter CDP). These facts suggest that a careful study must be made of possible mixing mechanisms in metal-poor stars in order to interpret the observed ^{7}Li abundances. Calculations by Pinsonneault, Deliyannis & Demarque (1992) suggest that rotational mixing may lead to a depletion of ^{7}Li by at least a factor of ten, in *all* metal-poor stars. Such a large change in the primordial ^{7}Li abundance could lead to an

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[4]In stars, Li may exist in two different isotopes: ^{6}Li and ^{7}Li. It is extremely difficult observationally to distinguish between ^{6}Li and ^{7}Li, thus the observations typically determine the total Li present in a star. Smith, Lambert & Nissen (1992) determined the abundance ratio of ^{6}Li/^{7}Li to be 0.05 ± 0.02 in the halo star HD 84937. It appears that the total Li content in a star is dominated by ^{7}Li. Thus, we will assume that the Li observations actually measure ^{7}Li.

incompatibility with standard BBN. This has far reaching consequences regarding the early history of our universe. However, the great uncertainty in the rotational mixing coefficients prevented Pinsonneault et $al.$ from making definitive conclusions regarding the primordial ^{7}Li abundance.

In addition to being an important probe of primordial nucleosynthesis, the abundance of ^{7}Li in the halo stars serves as another test of stellar evolution models. Chaboyer (1993) examined the issue of mixing in stellar radiative zones by comparing stellar evolution models to ^{7}Li abundances and rotation velocities in the Sun and young cluster stars with $M \leq 1.3\ M_\odot$. It was found that standard models, and models which included microscopic diffusion did not deplete enough ^{7}Li on the main sequence. Stellar models which included rotational mixing as well as diffusion were able to reproduce the observed ^{7}Li depletion pattern. These results are presented by CDP. The halo stars provide a severe test for our models, for a nearly uniform abundance of ^{7}Li is observed over a wide range in effective temperature ($5600 \lesssim T_{\rm eff} \lesssim 6450$) and metallicity, $-1.30 \lesssim$ [Fe/H] $\lesssim -3.80$ (hereafter referred to as the 'plateau'). Thus, any mixing mechanism which leads to the destruction of ^{7}Li must do so in a way which is independent of the stars temperature (mass) and metallicity.

The ^{7}Li depletion patterns in metal poor stellar models have been studied extensively by Deliyannis & Demarque (1991a,b) who included microscopic diffusion in an approximate manner, and Pinsonneault et $al.$ (1992) who considered the effects of rotational mixing. However, the interaction between diffusion and rotational mixing has not been studied. Proffitt & Michaud (1991) and Chaboyer et $al.$ (1992) studied the effect of microscopic diffusion on the primordial ^{7}Li abundance in some detail. Since these works were completed, the opacities have been updated (Iglesias & Rogers 1991; Kurucz 1991), new model atmospheres from Kurucz (1992) have become available and the energy producing nuclear reaction rates have been updated (Bahcall & Pinsonneault 1992). In addition, a large sample of extremely metal poor stars have had their ^{7}Li abundances determined by Thorburn (1994), who detected ^{7}Li in 78 stars with [Fe/H] < -1.90 and found upper limits in 6 more stars. Such a large sample of ^{7}Li abundances (which have been determined in a consistent manner) serve as an ideal test of our stellar models and BBN.

In this paper, we examine the question of the primordial ^{7}Li abundance by constructing stellar models with $Z = 10^{-4}$ and 10^{-5} ([Fe/H] $= -2.28$ and -3.28) and comparing them to the observations. This paper complements CDP, where the stellar models are compared to young clusters stars and the Sun, with $-0.10 \leq$ [Fe/H] $\leq +0.15$. A future paper will present the ^{6}Li and Be depletion isochrones, compare them to the observations, and comment on the constraints to BBN.

In §2. we present a short discussion of the new observations made by Thorburn (1994). The construction of the stellar models and the importance of the surface boundary conditions in determining the ^{7}Li depletion in cool stars is discussed in §3. We examine standard models in §4., models which include microscopic diffusion in §5. and models which include both rotational mixing and microscopic diffusion in §6. A discussion of the primordial ^{7}Li abundance and an overview of the major conclusions of this paper is presented in §7.

2. The Observations

Thorburn (1994) determined accurate ^{7}Li abundances for over 80 stars which have [Fe/H] < -1.9. The typical error in $\log N(Li)$ is ~ 0.1 dex, which is considerably smaller than previous work. The large number and high accuracy of these new observations reveal trends which were not discernible with the previous data. Three stars in the plateau star temperature range ($T_{\rm eff} > 5590$ K) are highly depleted. If these three stars are ignored, then a two dimensional fit to the plateau star observations reveals a significant trend of increasing $\log N(Li)$ with *both* $T_{\rm eff}$ and [Fe/H]. Thorburn determined slopes of $d \log N(Li)/d[Fe/H] = 0.134$ dex per dex and $d \log N(Li)/dT_{\rm eff} = 3.4 \times 10^{-4}$ dex per K. In addition, the data have a significant dispersion of 0.07 dex in $\log N(Li)$ about these straight line fits.

The trend with $T_{\rm eff}$ is not too surprising, for previous data had hinted that such a trend might exist. However, the trend of decreasing ^{7}Li abundance with decreasing metallicity is unexpected. This metallicity trend is present at a very high confidence level. In determining trends with [Fe/H] one must proceed with caution, due to possible systematic errors in the temperature scale or in the model atmospheres which might correlate with metallicity. If such errors are present, they could easily yield the observed trend of ^{7}Li abundance with [Fe/H]. If this trend is real, it has important ramifications on our interpretation of the data. Such a trend is naturally explained by the hypothesis that ^{7}Li production has occurred by galactic cosmic ray fusion (Thorburn 1994). As such, the initial abundance of ^{7}Li in the stars is not the primordial one, but has been enhanced due to the ^{7}Li production. The observed dispersion in $\log N(Li)$ is the result of a dispersion of 2 Gyr in the halo [Fe/H]–age relation.

An alternative explanation for the increase in $\log N(Li)$ with increasing [Fe/H] is that the ^{7}Li depletion in the models is larger for models with lower metallicity. However, as

is shown later, all of our models exhibit the opposite trend: the amount of ^{7}Li depletion decreases with decreasing metallicity.

Since the presence of the trend of increasing $\log N(Li)$ with increasing [Fe/H] has not been verified, we have decided to proceed with caution in interpreting the data. When determining if the models are a good fit to the plateau star observations, we have removed the [Fe/H] trend from the data by correcting all stars to [Fe/H] $= -3.0$ before fitting our ^{7}Li destruction isochrones to the observations in the T_{eff} plane. The isochrones are fit to the data using a χ^2 technique, where the only free parameter is the initial ^{7}Li abundance. Due to the real dispersion in the data, it is impossible to obtain a good fit with any low order function and so we have multiplied the error bars given by Thorburn by 1.3. When the error bars are increased by this amount, then the straight line fit given by Thorburn yields a reduced χ^2 of 0.917, which is a acceptable fit. In determining the initial ^{7}Li abundance implied by the models, we fit the ^{7}Li destruction isochrones to the raw data, using the true error bars. In performing our fits to the plateau data, we ignore the 3 highly depleted plateau stars which were found by Thorburn. The importance of the suggested positive correlation between ^{7}Li abundance and metallicity and of the highly depleted plateau stars will be discussed in §7.

3.   Model Construction & Surface Boundary Conditions

In order to construct metal poor ^{7}Li destruction isochrones, stellar models with masses ranging from 0.55 to 0.80 $M_\odot$ (in ~ 0.03 $M_\odot$ increments) and metallicities of $Z = 10^{-4}$ and 10^{-5} were evolved from the pre-main sequence to an age of 18 Gyr, or main sequence turnoff. Models which include no mixing in the radiative region (standard), models which include microscopic diffusion, and models which include both diffusion and rotational mixing (combined) were calculated. The prescription for mixing used in the models is fully described in Chaboyer (1993) and CDP, here we give a brief description. The microscopic diffusion coefficients are from Michaud & Proffitt (1993). The rotational mixing coefficients are taken to be the product of a velocity estimate and the radius (Zahn 1992). The velocity estimates are similar to those used by Pinsonneault *et al.* (1989). The rotating models include a general parameterization for the loss of angular momentum at the surface due to magnetic stellar winds. The formulation is similar to that given by Kawaler (1988), except that a saturation level, ω_{crit} has been introduced. If the surface angular velocity is below ω_{crit} then the rate of angular momentum loss is that given by Kawaler (1988). Above ω_{crit}, angular momentum loss occurs at a reduced level compared to that given by Kawaler (1988).

The physics in the models are identical to those used in Chaboyer (1993) and CDP. Specifically, the energy producing nuclear reaction rates are from Bahcall & Pinsonneault (1992); reaction rates for the ^{6}Li, ^{7}Li and ^{9}Be are from Caughlan & Fowler (1988); the high temperature opacities from Iglesias & Rogers (1991); the low temperature opacities (below 10^4 K) are from Kurucz (1991); while the surface boundary conditions are determined using Kurucz model atmospheres (Kurucz 1992; Howard, 1993). The fit between the stellar model and the atmosphere is made at an optical depth of $\tau = 2/3$. For temperatures above 10^6 K, a relativistic degenerate, fully ionized equation of state is use. Below 10^6 K, the single ionization of ^{1}H, the first ionization of the metals and both ionizations of ^{4}He are taken into account via the Saha equation. It was found by Chaboyer (1993) that these models gave an excellent description of the pre-main sequence ^{7}Li depletion in solar metallicity stars. Specifically, the ^{7}Li depletion isochrones were in good agreement with the Pleiades ^{7}Li data of Soderblom $et\ al.$ (1993) over the entire effective temperature range investigated ($6500 \le T_{\mathrm{eff}} \le 4000$ K).

In comparing to halo star ^{7}Li observations, we must pick an age for the stars. Unfortunately, it is impossible to determine ages of isolated field stars. We have found that globular clusters have ages ranging from 10 to 17 Gyr (Chaboyer, Sarajedini & Demarque 1992). It is likely that the metal poor halo stars and globular clusters formed at approximately the same time. If the hottest stars (in a given metallicity range) for which ^{7}Li observations have been obtained, are interpreted as being near the main sequence turn off, then a comparison to theoretical isochrones suggest an age of ~ 18 Gyr. The process of galaxy formation is not understood well enough to give us a definitive age for the halo stars. For this reason, we construct ^{7}Li isochrones for the ages 10, 14 and 18 Gyr.

The models were fit to the data by varying the initial ^{7}Li abundance until the best match to the observations was obtained. It was immediately clear that our models did not match the observed depletion in the cool stars ($T_{\mathrm{eff}} < 5300$ K). Virtually no depletion of ^{7}Li occurred in the standard models, or diffusion models. This is very surprising, as previous work had a significant depletion of ^{7}Li for low mass stars (Deliyannis & Demarque 1991a,b). The large amounts of ^{7}Li depletion observed in cool stars (and found by previous models) is due to the high temperatures and densities achieved at the base of the convection zone (particularly during the pre-main sequence) in low mass stars which lead to substantial ^{7}Li destruction. Although some ^{7}Li depletion occurred in the cool star models which included rotation and diffusion, it was at a level similar to the depletion suffered by the plateau stars. This problem with the models is shown in Figure 1, where both standard models, and combined models are compared to the observations.

To understand the source of this discrepancy, models were run with different opacities,

Fig. 1.— Comparison of standard and combined ^{7}Li destruction isochrones to the observations. The isochrones shown have an age of 18 Gyr. The initial ^{7}Li abundance was taken to be 2.25 (standard) and 3.0 (combined).

model atmospheres and mixing length. Changing from the OPAL opacities (Iglesias & Rogers 1991) to the LAOL opacities (Huebner *et al.* 1977) had very little effect on the models. Similarly, changing the low temperature opacities from Kurucz (1992) molecular opacities to Cox & Stewart (1970) opacities has only a minor effect on the models and their ^{7}Li depletion. Changing the mixing length within the stellar model (not the atmosphere) to 3.0 did not substantially alter the ^{7}Li depletion. It is important to realize that these statements are not true in general – usually the ^{7}Li depletion in cool stellar models is a sensitive function of the assumed physics. The reason that this is not true in this particular case, is that the temperature at the base of the convection zone in these models is far lower than that which is required to deplete ^{7}Li.

Large amounts of overshoot at the base of the surface convection zone lead to substantial ^{7}Li depletion on the pre-main sequence. We found that stellar models with an overshoot layer of ~ 0.3 pressure scale heights gave a reasonable match to the halo star ^{7}Li observations. However, such a large overshoot layer is clearly ruled out by the observed ^{7}Li depletion pattern in the Pleiades, whose stars have just arrived on the main sequence. Good agreement with the Pleiades ^{7}Li observations occurs when the overshoot layer has a depth of $H_p \lesssim 0.05$ pressure scale heights (CDP). For this reason, we do not consider large amounts of overshoot as an acceptable explanation of the discrepancy between the models

and observations.

Using a grey atmosphere (as opposed to a Kurucz atmosphere) for the surface boundary conditions had a profound effect on the cool models [5]. The grey atmospheres were calculated in the standard manner (Mihalas 1978). We note that grey atmospheres ignore the effects of convection. The Kurucz atmospheres treat convection using the mixing length approximation (with a mixing length of 1.25). The impact of the different model atmospheres on the temperature and density at the base of the convection zone in a $M = 0.55\ M_\odot$ stellar model (main sequence effective temperature ~ 4800 K). is shown in Figure 2. The models with Kurucz model atmospheres have somewhat thinner convection zones, which leads to lower temperatures and densities at the base of the convection zone. This in turn leads to substantially less burning of ^{7}Li. Observations indicate that this star should deplete ^{7}Li by > 2.2 dex as compared to the plateau stars. The model which uses the grey atmosphere depletes ^{7}Li by 2.3 dex, while the Kurucz model only depletes ^{7}Li by 0.25 dex. We see that the amount of ^{7}Li depletion in cool halo stars is extremely sensitive to the surface boundary conditions.

It is clear that models constructed using the grey atmosphere approximation are a better representation of the halo star ^{7}Li data then the Kurucz atmosphere models. Thus, we conclude that there could be a problem with the surface boundary conditions derived from metal-poor Kurucz atmospheres. We caution, however, that no direct observations of ZAMS Pop II ^{7}Li abundances exist. Thus, our models cannot be used to conclusively claim that the surface boundary conditions derived from metal-poor Kurucz atmospheres are in error. For example, there might exist a main sequence mixing mechanism which is more efficient in cooler stars. Such a mechanism could cause the models with Kurucz atmospheres to match the present ^{7}Li observations in halo stars (C. Proffitt, private communication). We note however, that no physical justification exists for such a ^{7}Li destruction mechanism, and such a ^{7}Li destruction mechanism does not exist in Pop I stars. For this reason, we have decided not to use the Kurucz model atmospheres in the remainder of this paper. Henceforth, all of the models presented have been evolved using a grey atmosphere, in contrast to the models presented by CDP, which used the Kurucz atmospheres. We note that the good fit which CDP found to the Pleiades ^{7}Li observations is only true when the Kurucz atmospheres are used – the use of grey atmospheres leads to too much ^{7}Li depletion

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[5]In doing this comparison, the mixing length of the models was kept fixed. This is a reasonable approximation, as a solar model which is evolved with a grey atmosphere but with the calibrated parameters from a Kurucz atmosphere run matches the observed solar radius and luminosity to within 1%.

Fig. 2.— Temperature and density at the base of the convection zone for a $M = 0.55 \ M_{\odot}$, $Z = 10^{-4}$ stellar model. Observations indicate that this star should deplete ^{7}Li by more than two orders of magnitude. The model which uses Kurucz (1992) model atmospheres to determine the surface boundary condition has convection zone base which is substantially cooler and less dense than the model which uses a grey atmosphere. The Kurucz model depletes ~ 0.25 dex of ^{7}Li, while the grey atmosphere model depletes the surface ^{7}Li by ~ 2.3 dex.

as compared to the Pleiades stars with $T_{\text{eff}} < 5500$ K. It is also important to note that the difference in ^{7}Li depletion between the Kurucz and grey atmosphere only matters for cool stars. The ^{7}Li depletion in the plateau stars are relatively insensitive to the particular choice of surface boundary conditions. Thus, the conclusions we draw in this paper regarding the ^{7}Li depletion in the plateau stars do not depend on our choice of surface boundary conditions.

4. Standard Models

^{7}Li destruction isochrones were constructed from stellar models which incorporated various types and amounts of mixing in the radiative region of the models. Specifically, we evolved a set of standard models (no mixing), two sets of models which included differing amounts of diffusion, and two sets of combined models which included differing amounts

TABLE 1
Stellar Model Parameters

					Rotation Parameters[b]			
Model	Mixing Length	Overshoot (H_p)	f_m[a]	Rotating?	N[c]	f_{GSF}[d]	f_μ[e]	ω_{crit}[f] $(10^{-5}s^{-1})$
UU	1.83825	0.05	1.0	Yes	2.0	10	0.10	3.0
VN	1.80600	0.02	0.8	Yes	1.5	1.0	0.01	1.5
KB	1.86200	0.02	1.0	No	—	—	—	—
ND	1.83493	0.02	0.8	No	—	—	—	—
LA	1.72800	0.02	0.0	No	—	—	—	—

[a]Constant which multiplies the microscopic diffusion coefficients of Michaud & Proffitt 1993.

[b]For a full description of the rotation parameters, see CDP.

[c]Power law index for the wind loss low.

[d]Constant which multiplies the estimate for the GSF diffusion coefficient.

[e]Constant which determines the efficiency of rotational mixing in the presence of mean molecular weight gradients. Small values of f_μ imply efficient mixing.

[f]Saturation level in the angular momentum loss law.

of rotational mixing and diffusion (combined models). The mixing parameters used in the stellar models presented in this paper are shown in Table 1. The different sets of models are referenced by a two letter code given in column 1 (the same code as was used to identify the models in CDP). Here, we comment on the ^{7}Li depletion in the standard models.

The low mass standard models deplete ^{7}Li on the main sequence. This results in more ^{7}Li depletion in older isochrones, as is demonstrated in Figure 3. Although the cool star fits are not perfect, it is clear that the older isochrones produce a better match to the observations. This is not unexpected, as is unlikely that the extremely metal poor stars shown here have an age of 10 Gyr. An isochrone with an age of ~ 22 Gyr would be a nearly ideal match to the observations. If the hottest stars (in a given metallicity range) are interpreted as being near the main sequence turn off, then a comparison to theoretical isochrones suggest an age of ~ 18 Gyr. Thus, it appears that the standard models do not produce the ^{7}Li depletion observed in cool stars. This might be due to uncertainties in the surface boundary conditions.

The dependence of the amount of ^{7}Li depletion on metallicity is shown in Figure 4. The $Z = 10^{-5}$ isochrone ([Fe/H] $= -3.28$) is completely flat between 6500 K and 5400

K. This is at variance with the observations. In addition, the $Z = 10^{-5}$ is considerably higher than the $Z = 10^{-4}$ isochrone below 5600 K. From the model standpoint, this is to be expected, because as we go to lower metallicities the convection zones become shallower and it is expected that less ^{7}Li burning will occur. However the few cool star observations that are available do not show such a dramatic trend with metallicity. It is interesting to note that Deliyannis & Demarque (1991b) found the opposite trend with metallicity. Their metal poor models depleted more ^{7}Li than their metal rich models due to a nonlinear dependence of the depth of the surface convection zone on metallicity. It is unclear why their models have this property. We are using identical ^{7}Li nuclear reaction rates. The major difference between this work and Deliyannis & Demarque is that we are using the OPAL (Iglesias & Rogers 1991) and Kurucz (1992) opacities, while they used the Cox & Stewart (1970) opacities.

From Figures 3 and 4 we see that the best fit to the data is obtained using the $Z = 10^{-4}$ 18 Gyr isochrone. This isochrone still does a poor job of matching the cool stars, but the fit to the plateau stars is quite acceptable with a reduced χ^2 of 1.15, which is well within the 2σ range. The primordial abundance inferred from this isochrones is $\log N(Li) = 2.25 \pm 0.012$. Many of the other $Z = 10^{-4}$ isochrones produce acceptable fits to the date, and yield primordial ^{7}Li abundances in the narrow range $2.22 \leq \log N(Li) \leq 2.26$. However, the $Z = 10^{-5}$ isochrones are perfectly flat over the plateau temperature range. A fit of these isochrones to all of the plateau observations is rejected by the χ^2 analysis. This suggests the fact that the ^{7}Li depletion in our standard models have too great of a dependence on the metallicity. If we restrict our attention to stars with $[Fe/H] \leq -2.80$ (of which there are only 21 stars), then the $[Fe/H] = -3.27$ are an acceptable fit. In performing this fit to the extremely metal poor stars, we have multiplied the error bars by 1.1 (and continue to correct for the $[Fe/H]$ trend), to allow for the dispersion at a given $T_{\rm eff}$. The fit is shown graphically in Figure 5 where we compare the best fitting $Z = 10^{-5}$ isochrone to the extremely metal poor plateau stars. It is clear from Figure 5 that there are not enough observations of extremely metal poor stars to rule out the possibility the flat isochrones produced by standard models with $Z = 10^{-5}$. However, it is likely that the the trend of increasing $\log N(Li)$ with increasing $T_{\rm eff}$ is true for these stars as well. It appears that the standards models do a poor job of matching the observations.

Fig. 3.— Comparison of 10, 14 and 18 Gyr ^{7}Li standard (LA) isochrones with $Z = 10^{-4}$ to the observations. The initial ^{7}Li abundance was taken to be 2.23.

Fig. 4.— Comparison of $Z = 10^{-4}$ and 10^{-5} 18 Gyr ^{7}Li standard (LA) isochrones to the halo star observations. The initial ^{7}Li abundance was taken to be 2.25.

Fig. 5.— A [Fe/H] $= -3.27$ ^{7}Li standard (LA) destruction isochrone is compared to observations of $\log \mathrm{N(Li)}$ in stars with [Fe/H] ≤ -2.8. A χ^2 fit reveals that the isochrone is acceptable at the 2σ level. The initial ^{7}Li abundance was taken to be 2.20.

Fig. 6.— 18 Gyr ND ^{7}Li destruction isochrones are compared to the observations. The diffusion coefficients have been multiplied by 0.8, but the isochrones still have too much curvature and are rejected by a χ^2 analysis. The initial ^{7}Li abundance as determined by the χ^2 analysis was 2.40 for $Z = 10^{-4}$ and 2.45 for $Z = 10^{-5}$.

5. Pure Diffusion Models

Microscopic diffusion causes ^{7}Li to settle out of the surface convection zone. The time scale for this settling proportional to the mass of the surface convection zone. The mass of the surface convection zone is a function of mass and metallicity with the mass of the surface convection zone decreasing for high mass and/or low metallicity stars. Thus, uninhibited microscopic diffusion will lead to a downward curvature of the ^{7}Li destruction isochrones at the hot edge of the plateau. The observational database available prior to Thorburn (1994) did not rule out such a trend. As such, previous studies of the effects of microscopic diffusion on halo star ^{7}Li abundances obtained satisfactory agreement with the observations (Proffitt & Michaud 1991; Chaboyer *et al.* 1992).

The new data rule out any trend of decreasing $\log N(\text{Li})$ with increasing T_{eff} in the plateau temperature range ($T_{\text{eff}} > 5590$ K). This is shown graphically in Figure 6 where we plot our best fitting pure diffusion models to the observations. Clearly, these isochrones do not reproduce the mean trend visible in the data. A χ^2 fit of the diffusive isochrones to the plateau data rejects all of the isochrones at a very high probability. The results of the χ^2 analysis are presented in Table 2 from which it is clear that diffusive isochrones do not

TABLE 2
χ^2 Fits of the Diffusive Isochrones to Plateau Stars

Isochrones			Observational	Reduced		Initial ^{7}Li
Age (Gyr)	[Fe/H]	$f_{\rm m}$[a]	Data Set	χ^2	Probability[a]	Abundance
10	−2.28	1.0	all stars	1.86	< 0.001	2.36
14	−2.28	1.0	all stars	1.84	< 0.001	2.40
18	−2.28	1.0	all stars	1.57	< 0.01	2.44
10	−2.28	0.8	all stars	1.75	< 0.001	2.34
14	−2.28	0.8	all stars	1.73	< 0.001	2.37
18	−2.28	0.8	all stars	1.49	< 0.01	2.40
14	−2.28	0.8	[Fe/H] > −2.8	1.86	< 0.001	2.37
18	−2.28	0.8	[Fe/H] > −2.8	1.57	< 0.001	2.41
14	−3.28	0.8	all stars	2.59	< 0.001	2.42
18	−3.28	0.8	all stars	1.64	< 0.001	2.45
10	−3.28	0.8	[Fe/H] < −2.8	2.12	< 0.01	2.39
14	−3.28	0.8	[Fe/H] < −2.8	2.32	< 0.001	2.44
18	−3.28	0.8	[Fe/H] < −2.8	2.31	< 0.001	2.46

[a]Constant which multiplies the microscopic diffusion coefficients of Michaud & Proffitt 1993.
[b]Probability of a random data set exceeding the reduced χ^2 when compared to the isochrone.

match the observations. We have also performed a χ^2 analysis of the diffusive isochrones which use the Kurucz (1992) model atmospheres as surface boundary conditions, and find that they too are rejected as good fits to the log N(Li) data with $T_{\rm eff} > 5590$ K. We conclude that the diffusion coefficients given by Michaud & Proffitt (1993) yield too much depletion of ^{7}Li for the hot stars. The models shown in Figure 6 have had their diffusion coefficients artificially lowered by 25%, which demonstrates that even an error of $\sim 25\%$ in the diffusion coefficients does not effect our conclusion that diffusion leads to an over depletion of ^{7}Li in the hot stars.

6.   Combined Rotation and Diffusion Models

Rotational mixing in the models can lead to the inhibition of microscopic diffusion near the surface. Thus, it is likely that the ^{7}Li depletion isochrones for the combined models will

not possess the strong downward curvature with increasing T_{eff} that is observed in pure diffusion models. Previous studies of the impact of rotational mixing (without diffusion) on halo stars ^{7}Li abundances found a rather large, uniform depletion (Pinsonneault *et al.* 1992) and so it is important to study the combined effect of both mixing processes. Chaboyer (1993) determined that the model VN parameters (see Table 1) provided the best match to the ^{7}Li observations in the Sun and young cluster stars. For this reason, we have chosen to evolve a set of low metallicity models using the same VN parameters (except for the use of a grey atmosphere). The initial rotational velocity of the models is a free parameters, we have evolved models with $V_{rot} = 10$, 30 and 50 km/s, which encompasses the range observed in T-Tauri stars (Bouvier *et al.* 1993). The $Z = 10^{-4}$ isochrones provide a good match to the observations and are shown in Figure 7. The 14 Gyr observations show a slight downward trend at the hot edge of the plateau which is not present in the 18 Gyr isochrones. The χ^2 fit to the plateau stars reveals that the 14 and 18 Gyr VN isochrones are a good match to the observations. The initial ^{7}Li abundance required to match the observations is $\log N(\text{Li}) = 3.03$ and $\log N(\text{Li}) = 3.13$ for the 14 and 18 Gyr isochrones respectively. These values are similar to those found by Pinsonneault *et al.* (1992) for pure rotation models. In contrast to the standard models presented in §4., these combined isochrones do a good job of fitting the observations of ^{7}Li in the cool stars ($T_{\text{eff}} < 5500$ K).

An examination of the time and mass dependence of the ^{7}Li depletion reveals the reason for the flattening of the isochrones at 18 Gyr. The ^{7}Li depletion as a function of age is plotted in Figure 8 for stars with masses of 0.75, 0.72 and 0.70 $M_\odot$. These stars lie on the hot half of the plateau and so are responsible for the flattening of the isochrones. During the early pre-main sequence ($t < 10$ Myr) all models experience a modest amount of surface ^{7}Li depletion. Low mass stars deplete somewhat more ^{7}Li. Starting near the ZAMS, ($t \sim 60$ Myr) rotational mixing depletes the surface ^{7}Li abundance. This depletion is more efficient in the lower mass stars. The lower mass star have deeper convection zones, hence the ^{7}Li needs to be transported a smaller distance before being destroyed. After ~ 6 Gyr the rotational mixing time scale is very long and the depletion due to diffusion becomes important. The diffusive ^{7}Li depletion time scale is proportional to the mass of the surface convection zone. The convective envelope mass (and hence, the time scale for ^{7}Li depletion) declines on the main sequence. A $Z = 10^{-4}$, $M = 0.75$ $M_\odot$ stellar model has a convective envelope mass less than 4×10^{-3} $M_\odot$ at 6 Gyr. At this point the mixing due to diffusion causes the ^{7}Li surface depletion rate to be significantly higher in a 0.75 $M_\odot$ model than a 0.72 $M_\odot$ model. At 11 Gyr, the surface ^{7}Li abundance is very similar in these two models. By 14 Gyr, the surface ^{7}Li abundance of the 0.75 $M_\odot$ model is well below that of the 0.72 $M_\odot$ model, and so the ^{7}Li destruction isochrone shows a downward curvature at 14 Gyr. The 0.75 $M_\odot$ model has a turnoff age of ~ 15 Gyr, and so does not contribute to our

Fig. 7.— Model VN isochrones with ages of 14 and 18 Gyr are compared to the observations. For each age, two isochrones are shown, corresponding to initial rotation velocities of 10 and 30 km/s on the pre-main sequence. The 30 km/s isochrones are ~ 0.3 dex below the 10 km/s isochrones for $T_{\mathrm{eff}} > 5590$ K. The initial ^{7}Li abundance determined by a χ^2 fit to the plateau stars is 3.03 for the 14 Gyr, 10 km/s isochrone and 3.13 for the 18 Gyr, 10 km/s isochrone.

Fig. 8.— ^{7}Li depletion as a function of age for $Z = 10^{-4}$ combined models.

18 Gyr ^{7}Li destruction isochrone. Instead, the hot edge of the 18 Gyr isochrone is located by the 0.72 $M_\odot$ model whose convection zone depth is considerable larger than the 0.75 $M_\odot$ model. The diffusion time scale is such that by an age of 18 Gyr, the total ^{7}Li depletion in a 0.72 $M_\odot$ model is similar to a 0.70 $M_\odot$ model leading to a ^{7}Li destruction isochrone which does not turn down at hot temperatures.

Over the plateau star $T_{\rm eff}$ range ($T_{\rm eff} \gtrsim 5600$ K), the $V_i = 30$ km/s isochrones are depleted ~ 0.3 dex more than the 10 km/s isochrones. Since stars are likely to have a range of initial rotation velocities, this implies that the combined models predict a dispersion in the observed ^{7}Li abundance. It is impossible to determine the initial rotation velocity distribution for halo stars. Observations of rotation periods in low mass T-Tauri stars indicate that 2 out of 19 single stars (10%) have rotation velocities greater than or equal to 30 km/s (Bouvier 1991). No stars were found to rotate slower than 10 km/s (but there is an observational bias against long periods) while 11 stars (58%) had rotation velocities in the range 10 – 20 km/s. We have run a few models with $V_i = 20$ km/s which indicate that these stars experience ~ 0.2 dex more depletion than the 10 km/s models. Assuming that the halo stars have a similar distribution in initial rotation velocities as the T-Tauri stars, the above numbers suggest that our rotation models should have a dispersion of ~ 0.15 dex, which is similar to the 0.11 dex dispersion present in the observations before the metallicity trend is removed. However, after the metallicity trend is removed, the dispersion in the observations is 0.07 dex and it would appear that the models predict too large of a dispersion. A more careful study of predicted dispersion is required before making any definitive conclusions.

The isochrones change dramatically for the $Z = 10^{-5}$ models. At a given $T_{\rm eff}$, the convection zone depths in these models are shallower than the $Z = 10^{-4}$ models. Hence, microscopic diffusion plays a more important role in these models. This is shown in Figure 9 where the 10, 14 and 18 Gyr $Z = 10^{-5}$ isochrones are plotted. The ^{7}Li depletion in these plateau temperature range is approximately 0.1 dex *less than* that found in the $Z = 10^{-4}$ isochrones. This is in the opposite direction to the $\log N({\rm Li})$ – [Fe/H] trend observed in the data. In addition, there is a large downward trend present in all of these isochrones which is not present in the observations. A χ^2 analysis rejects these fits at the greater than 3σ level. A fit to those stars with [Fe/H] ≤ -2.8 also rejects these isochrones at a greater than 3σ level. This is graphically illustrated by Figure 10 where the metal poor isochrones are compared to the stars which have [Fe/H] ≤ -2.8 and $T_{\rm eff} > 5700$ K. This clearly demonstrates that the only parameters which gave a good representation of the ^{7}Li depletion pattern observed in young clusters fail to match the extremely metal poor halo star observations.

Fig. 9.— Model VN isochrones with ages of 10, 14 and 18 Gyr and $Z = 10^{-5}$ are compared to the observations. The $V_i = 10$ km/s isochrones are shown. The 30 km/s isochrones are ~ 0.3 dex below the 10 km/s isochrones for $T_{\text{eff}} > 5590$ K. The initial ^{7}Li abundance was taken to be 2.87.

Fig. 10.— Model VN isochrones with ages of 10, and 14 Gyr and [Fe/H] = -3.28 are compared to observations of stars which have [Fe/H] ≤ -2.8 and $T_{\text{eff}} > 5700$ K. The $V_i = 10$ km/s isochrones are shown. The initial ^{7}Li abundance was taken to be 2.87.

Fig. 11.— Model UU isochrones with ages of 10, 14 and 18 Gyr and $Z = 10^{-4}$ are compared to the observations. The $V_i = 10$ km/s isochrones are shown. The 30 km/s isochrones are ~ 0.25 dex below the 10 km/s isochrones for $T_{\text{eff}} > 5590$ K. The initial ^{7}Li abundance was taken to be 3.02

We have also constructed halo star models using model UU parameters from Table 1 in order to test the sensitivity of our conclusions to the rotational mixing parameters. The rotational mixing is much more sensitive to gradients in the mean molecular weight in these models as compared to the VN models presented previously. The $Z = 10^{-4}$ model UU isochrones are shown in Figure 11. Since diffusion of ^{4}He out of the surface convection zone leads to a large mean molecular weight gradient at the base of the convection zone, the rotational mixing is overwhelmed by the diffusion at a much earlier age than in the VN models. This is clear from the isochrones in Figure 11, all of which have some downward curvature to them over the plateau effective temperature range. The χ^2 analysis reject the 10 and 14 Gyr isochrones as acceptable fits to the data. The 18 Gyr isochrone has less curvature and is an acceptable fit to the observations at the 2σ level. It is clear that the VN isochrones are a better match to the data, which strengthens the CDP conclusion that the VN parameters provide the best fit to the observations. The primordial ^{7}Li abundance determined by the fit is $\log N(\text{Li}) = 3.02$, which is similar to that inferred from the 14 Gyr VN isochrone.

7. Summary

It is clear that the halo star ^{7}Li observations of Thorburn (1994) provide an excellent database with which to test stellar evolution models. Stellar evolution models which use the Kurucz (1992) model atmospheres as the surface boundary condition deplete virtually no ^{7}Li over the entire temperature range studied ($6500 < T_{\mathrm{eff}} < 4800$ K). This is clearly at odds with the observations, which show substantial ^{7}Li depletion below $T_{\mathrm{eff}} = 5600$ K. We conducted extensive tests which determined that the models which used grey atmospheres did not suffer from this problem. Changing the opacities or mixing length did not substantially alter the ^{7}Li depletion pattern. However, models which use the grey atmosphere approximation as the surface boundary condition provide a good match to the observations. It appears that the low metallicity Kurucz (1992) atmospheres do not provide the appropriate boundary conditions for our stellar evolution models. This is in sharp contrast to the solar metallicity models studied by CDP, where the use of the Kurucz atmospheres, and not the grey atmospheres, provided an excellent match to the observations. We caution, however, that no direct observations of ZAMS Pop II ^{7}Li abundances exist. Thus, our models cannot be used to conclusively claim that the surface boundary conditions derived from metal-poor Kurucz atmospheres are in error.

Standard models (with Kurucz or grey atmospheres) and [Fe/H] $= -2.28$ do a good job of fitting the observed ^{7}Li abundances in stars hotter than ~ 5600 K. They predict a primordial ^{7}Li abundance of $\log \mathrm{N(Li)} = 2.24 \pm 0.03$. The cooler stars have somewhat lower abundances than the isochrones predict. This is not too serious a problem since the amount of depletion in the low mass models is a strong function of the amount of overshoot assumed at the base of the convection zone, and of the atmospheres used to calculate the surface boundary conditions. Of greater concern is the fact that decreasing the metallicity in the models leads to a decrease in the amount of ^{7}Li depletion. Although there are not many stars with [Fe/H] < -2.8 it appears that the observations do not possess this property.

The observations provide a very strict test of models which include microscopic diffusion, for these models predict a downward curvature in the ^{7}Li destruction isochrones at hot temperatures, regardless of the choice of choice of atmospheres. Even allowing for a dispersion in the initial ^{7}Li abundance, the observations clearly rule out models which include uninhibited microscopic diffusion of ^{7}Li from the surface of the star. This has been believed to have important implications for the diffusion of ^{4}He, for the time scale for the diffusion is very similar for ^{4}He and ^{7}Li (Chaboyer *et al.* 1992). However, the situation is different when rotation is considered. The rotational mixing in our models causes an order of magnitude depletion in ^{7}Li, while diffusion at the hot end of the plateau depletes ^{7}Li by a factor of ~ 2. Thus, the large overall depletion of ^{7}Li caused by rotational mixing over the

entire effective temperature range of the ^{7}Li plateau tends to overwhelm the ^{7}Li depletion caused by diffusion in the hot plateau stars. By contrast, ^{4}He is not depleted by rotational mixing, and so it is difficult to make statements regarding the depletion of ^{4}He, based on the constraints from ^{7}Li observations. For example, an 0.72 M$_\odot$ stellar model, at an age of 16 Gyr has a surface ^{4}He mass fraction 0.230 (standard, model LA); 0.141 (pure diffusion, model ND) and 0.154 (combined, model VN). The effective temperature of this model is 6292 K (standard), 6280 K (diffusion) and 6288 K (combined). Thus, we see that inhibition of diffusion by rotational mixing in hot halo stars is a relatively minor effect, reducing the surface depletion of ^{4}He by $\sim 10\%$. However, the shape of the ^{7}Li depletion isochrones are substantial altered by the presence of rotational mixing, leading to good agreement with the observations.

The diffusion of ^{4}He from the surface of the star has a dramatic effect on effective temperature of the stellar evolution model. In a previous paper, we found that uninhibited diffusion of ^{4}He from the surface altered the shape of our isochrones (in the M_V, B $-$ V plane) such that fits to globular cluster colour magnitude diagrams suggested an age reduction of $2 - 3$ Gyr compared to standard isochrones (Chaboyer *et al.* 1992). However, the age reduction derived using the $\Delta V(\mathrm{TO - HB})$ technique (which relies on the luminosity of the models, *not the effective temperature*) was very small, ~ 0.5 Gyr. In order to see if these conclusions are altered by the present calculations, we have calculated isochrones, in a similar manner to Chaboyer *et al.* (1992). Sample isochrones, for an age of 16 Gyr, are shown in Figure 12. We see that the introduction of rotational mixing does not substantial alter the shape of the isochrone (ie – the pure diffusion isochrone is quite similar to the isochrone which includes rotation and diffusion). We find the following turnoff magnitudes and colours for the 16 Gyr isochrone: M$_v$ = 3.78, B $-$ V = 0.341 (standard); M$_v$ = 3.86, B $-$ V = 0.362 (diffusion); and M$_v$ = 3.85, B $-$ V = 0.360 (combined). Hence, these calculations do not alter the conclusions of Chaboyer *et al.* (1992) regarding the effect of diffusion on age estimates of globular clusters. Figure 12 demonstrates that even in the presence of rotation, the dominant effect of diffusion is on the colours of the isochrones. Hence, age determination techniques which rely on using colours of the isochrones (such as isochrone fitting) imply that diffusion lowers the age estimate of globular clusters by 2 – 3 Gyr. In contrast, if we were to date globular clusters using the main sequence turn off luminosity, and an independent distance estimate, we would find an age reduction of 1 Gyr. The age reduction derived using the $\Delta V(\mathrm{TO - HB})$ technique is 0.5 Gyr, for this age determination relies on calculating the horizontal branch luminosity, which is also slightly lowered by diffusion.

Rotational mixing can inhibit the microscopic diffusion leading to less curvature in the ^{7}Li depletion isochrones for $T_{\mathrm{eff}} > 5600$ K. The [Fe/H] $= -2.28$ VN stellar models which

Fig. 12.— Comparison of 16 Gyr isochrones, which were calculated from stellar models with different mixing prescriptions. Shown are standard (model LA, dash-dot line), pure diffusion (model ND, dashed line) and combined, rotation and diffusion (model VN, solid line) isochrones. The combined model is quite similar to the pure diffusion model.

include both diffusion and rotational mixing provide an excellent match to the mean trend in $T_{\rm eff}$ which is present in the observations. Both the plateau stars and the heavily depleted cool stars are well fit by these models. The dispersion at a give $T_{\rm eff}$ predicted by the models due to varying initial rotation velocities is roughly consistent with what is observed. The rotational mixing leads to considerable ^{7}Li depletion in these models and the primordial ^{7}Li abundance inferred from these models is $\log {\rm N(Li)} = 3.08$, with an uncertainty of ~ 0.1 dex due to uncertainties in the rotational mixing coefficients. This is similar to the value found by Pinsonneault *et al.* (1992) for models which included rotational mixing, but not microscopic diffusion.

However, the $[{\rm Fe/H}] = -3.28$ isochrones reveal problems with the combined models. These isochrones predict a trend of decreasing $\log {\rm N(Li)}$ with increasing $T_{\rm eff}$ which is clearly not present in the observations. This indicates that the diffusion (which becomes more efficient at depleting ^{7}Li in hot stars as the metallicity decreases) is not being inhibited enough by the rotational mixing. Thus suggests that the rotational mixing is more efficient (particularly in very low metallicity stars) than has been modeled here. Also troubling is the fact that models with low metallicities deplete *less* ^{7}Li than the more metal rich models, for the observations of plateau stars reveal that the ^{7}Li abundance *decreases* with lower

metallicities. The positive correlation of $\log N(Li)$ with [Fe/H] is not compatible with these models unless ^7Li production has occurred.

Pure diffusion models are ruled out by ^7Li observations in young cluster stars and the Sun (CDP) and, as was shown in this paper, by the halo star ^7Li observations. The standard models provide a reasonable fit to the halo star ^7Li observations, but, as was shown by CDP, fail to account for the ^7Li depletion observed in the Sun, or young cluster stars. The combined models with rotational mixing and diffusion are a good match ^7Li observations in stars with [Fe/H] $\gtrsim -2.3$, but do not agree with the most metal poor halo star observations. It is clear that the observed ^7Li abundances provide a powerful test of stellar evolution models; a test which some of our models do not pass. The reason for this breakdown of our lowest metallicity combined models is uncertain. It could be that rotational mixing is more efficient in metal poor stars than is indicated by our prescription. Another possibility may be related to the surface boundary conditions in the models. The ^7Li depletion in the models can depend sensitively on the detailed atmospheric structure used in the models. We note that the effects of convection, which is not well understood, become increasingly important in low metallicity atmospheres.

In addition none of our models (standard, diffusive or combined rotation and diffusion) predict a decrease in $\log N(Li)$ with decreasing [Fe/H] which is present in the observations (Thorburn 1994). This trend is compatible with the ideal that ^7Li production has occurred due to galactic cosmic ray spallation (Thorburn 1994). If this is true, then the primordial ^7Li abundance must be lower than that obtained from fits to all of the plateau stars. If ^7Li production has occurred, then the observed dispersion in $\log N(Li)$ is easily explained by an age range of ~ 2 Gyr in stars with the same metallicity. We caution however, that the ^7Li abundances were determined using the Kurucz model atmospheres, which include the effects of convection using the mixing length approximation. Convection becomes increasingly vigorous in more metal-poor stars and a better understanding of atmospheric models is required to answer the question of decreasing ^7Li abundance with decreasing metallicity.

The presence of three highly depleted 'plateau' stars demonstrate that substantial ^7Li depletion is occurring in some plateau stars. Standard stellar evolution models do not predict any depletion in this temperature range. Young cluster observations clearly demonstrate that ^7Li destruction is occurring on the main sequence at a rate much larger than that predicted by standard models. This clearly points to the existence of an additional mixing mechanism in the radiative zones of some stars. If this mixing mechanism is related to rotation, our prescription for rotational mixing fails to match these observations. The understanding of mixing in the radiative zones of stars remains one of the outstanding

problems in stellar evolution theory.

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